

---

**NUMERICAL ALGORITHMS N4SID FOR SYSTEM IDENTIFICATION OF BUILDINGS**

---

**Sertaç Tuhta, Ibrahim Alameri, Furkan Günday**

Ondokuz Mayıs University,  
Faculty of Engineering,  
Department of Civil Engineering  
Samsun, Turkey

---

**ABSTRACT**

In this paper, a new structural identification tool is proposed to identify the modal properties of structures. At last, after collecting modal responses from the available sensors, the mode shape vector for each of the decomposed modes in the system is identified from all obtained modal response data. To demonstrate the efficiency of the algorithm, a series of numerical, and laboratory studies were evaluated. The laboratory case study utilized the vibration response of a steel model structures. The modal properties of the steel model were computed using analytical approach for a comparison with the experimental modal frequencies. Results demonstrated that N4SID system identification method is efficient and accurate in identifying modal data of the structures.

**KEYWORDS:** N4SID, System Identification, Modal data identification, Numerical Algorithms

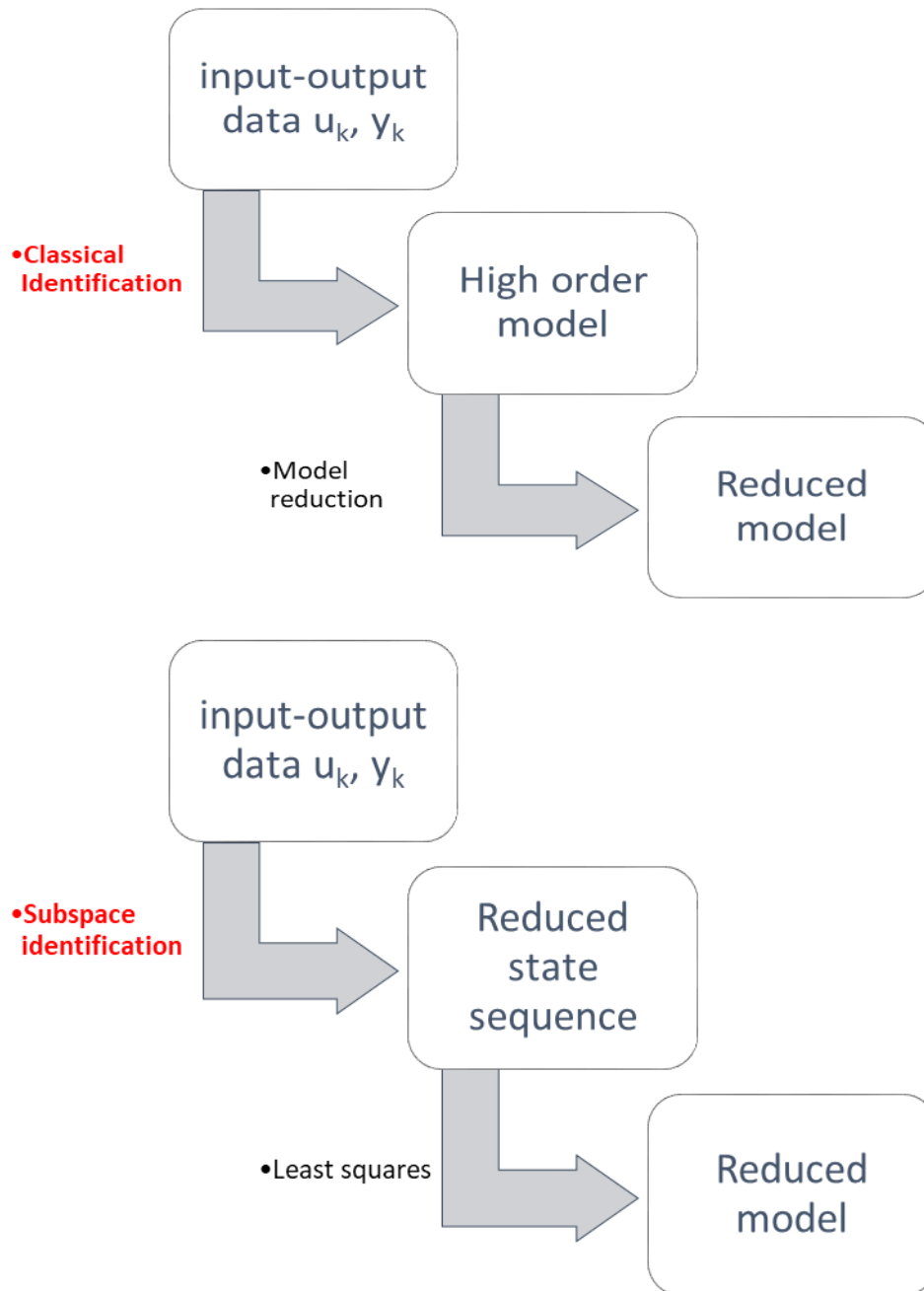
---

**INTRODUCTION**

System identification (SI) is a modeling process for an unknown system based on a set of input outputs and is used in various engineering fields. (Sirca and Adeli, 2012). Subspace system identification is introduced as a powerful black-box system identification tool for structures. The application of the method for supporting excited structures is emphasized in particular. The black- box state- space models derived from the identification of subspace systems are used to estimate the modal properties (i.e. modal damping, modal frequency and mode shapes) of the structures (Kim, 2011).

In engineering structures, three types of identification are used: modal identification of parameters; structural-modal identification of parameters; control model identification methods. In the frequency domain the identification is based on the unique value decomposition of the spectral density matrix and it is denoted Frequency Domain Decomposition (FDD) and its further development Enhanced Frequency Domain Decomposition (EFDD) (Kasimzade and Tuhta, 2012).

In the time domain there are three different implementations of the Stochastic Subspace Identification (SSI) technique: Unweight Principal Component (UPC); Principal component (PC); Canonical Variety Analysis (CVA) are used (Kasimzade and Tuhta, 2012).



**Figure 1** System identification aims to create input-output data state space models

When a reduced order model is required, one first identifies a high order model in some classical approaches (on the right) and then applies a model reduction technique to obtain a low order model. The left side shows the subspace identification approach: first we obtain a "reduced" status sequence, after which a low order model can be identified directly. (Overschee and Moor, 1996).

In this paper, the problem of multiple degrees of free structural systems without a limited number of elements was investigated. As known for similar type systems the system matrices  $[m]$ ,  $[c]$ ,  $[k]$  may be built only by FEM and the equation of motion for a finite-dimensional linear-dynamic system a set of  $n_2$  second-order differential equations are arranged as

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = [d]\{f_{\oplus}(t)\} \quad (1)$$

Here the direct stiffness method was used for implementation in the finite element method and appropriately was build system mass, damping and stiffness matrices ( $[m]$ ;  $[c]$ ;  $[k]$ ). For example The FEM implementing system stiffness matrix  $[k]$  is shown as follows by the direct stiffness method:

$$[\bar{k}_r] \rightarrow [\bar{k}_r] = [C_r][\bar{k}_r][C_r]^T \rightarrow [\bar{k}_{r+}] = [\tau_r]^T [\bar{k}_r] [\tau_r] \rightarrow [k_*] = \sum_{r=1}^{r_*} [\bar{k}_{r+}] \rightarrow a.b.c. \rightarrow [k] \quad (1b)$$

where,  $[\bar{k}_r]$  is the element stiffness matrix in local coordinate system (c.s.) for  $r$ -th finite element,  $[\bar{k}_{r+}]$  is the element stiffness matrix in global coordinate system for  $r$ -th finite element,

$[C_r]$  is the coordinate transformation matrix from local to global c.s. for  $r$ -th finite element,

$[\tau_r]$  is the topology matrix for  $r$ -th finite element,  $a.b.c.$  is abbreviation "mean after application of boundary conditions",  $r_*$  is a number of identical finite elements examined system,

$[k]$  is the stiffness matrix of the in examined system in global c.s.

The main relationships of the FEM are based on the Lagrange principle of variation.

The equation of motion (1) are transformed to the state-space former of first order equations- i.e., a continuous-time state-space model of the system are evaluated as

$$\{\dot{z}(t)\} = [A_c]\{z(t)\} + [B_c]\{f_{\oplus}(t)\} \quad (2a)$$

$$[A_c] = \begin{bmatrix} [0] & [I] \\ -[m]^{-1}[k] & -[m]^{-1}[c] \end{bmatrix} \quad (2b)$$

$$[B_c] = \begin{bmatrix} [0] \\ [m]^{-1}[d] \end{bmatrix} \quad (2c)$$

$$\{z(t)\} = \begin{bmatrix} u(t) \\ \dot{u}(t) \end{bmatrix} \quad (2d)$$

If the response of the dynamic system is measured by the  $m_1$  output quantities in the output vector  $\{y(t)\}$  using sensors (such as accelerometers, velocity, displacements, etc.), for system model represented by the equations (2), appropriate measurement-output equation become as

$$\{y(t)\} = [C_a]\{\ddot{u}\} + [C_v]\{\dot{u}\} + [C_d]\{u\} = [C]\{z(t)\} + [D]\{f_{\oplus}(t)\} \quad (3a)$$

$$[C] = [ [C_d] - [C_a][m]^{-1}[k] \quad [C_v] - [C_a][m]^{-1}[c] ] \quad (3b)$$

$$[D] = [C_a][m]^{-1}[d] \quad (3c)$$

Where  $\{u\}$  is the vector of displacement;  $[A_c]$ , is an  $n_1$  ( $n_1 = 2n_2$ ;  $n_2$  is the number of in depended coordinates) by  $n_1$  state matrix ;  $[d]$  is an  $n_2$  by  $r_1$  input influence matrix, characterizing the locations and type of known inputs  $\{f_{\oplus}(t)\}$ ;  $[C_a]$ ,  $[C_v]$ ,  $[C_d]$  are output influence matrices for acceleration, velocity, displacement for using sensors (such as accelerometers, tachometers, strain gages, etc.) respectively;  $[C]$  is an  $m_1 \times n_1$  output influence matrix for the state vector  $\{z\}$  and displacement only;  $[D]$  is an  $m_1 \times r_1$  direct transmission matrix;  $r_1$  is the number of inputs;  $m_1$  is the number of outputs.

In the output-only modal analysis environment, the main assumption is that input force  $\{F(t)\} = [d]\{f_{\oplus}(t)\}$  comes from white noise or time impulse excitation. Under this hypothesis discrete-time stochastic state {space model may be written as:

$$\{z_{k+1}\} = [A]\{z_k\} + [B]\{f_{\oplus k}\} + \{w_k\} \quad (4)$$

$$\{y_k\} = [C]\{z_k\} + [D]\{f_{\oplus k}\} + \{v_k\} \quad (5)$$

where  $\{z_k\} = \{z(k\Delta t)\}$  is the discrete-time state vector;  $\{w_k\}$  is the process noise due to disturbance and modeling imperfections;  $\{v_k\}$  is the measurement noise due to sensors' inaccuracies;  $\{w_k\}, \{v_k\}$  vectors are non-measurable, but assumed that they are white noise with zero mean.

If this white noise assumption is violated, in other words if the input contains also some dominant frequency components in addition to white noise, these frequency components cannot be separated from the Eigen frequencies of the system and they will appear as Eigen values of the system matrix  $[A]$ .

In the real structures, excited by ambient vibration, the input  $\{f_{\oplus}(t)\}, \{f_{\oplus k}\}$  remains unmeasured and therefore it disappears from the equations (2)-(5) respectively. Then to take into consideration this fact, the input is implicitly modeled by the noise terms  $\{w_k\}, \{v_k\}$ , which are indirectly contain no measurable input from ambient vibration and mentioned relation became as:

$$\{z_{k+1}\} = [A]\{z_k\} + \{w_k\} \quad (6)$$

$$\{y_k\} = [C]\{z_k\} + \{v_k\} \quad (7)$$

## METHODOLOGY

### Description of Model Steel Structure:

The Quanser shake table II is a uniaxial bench-scale shake table. The shake table unit can be controlled by appropriate software as illustrated in Figs. 2a, b. It is effective for a wide variety of experiments for civil engineering structures. Shake table specifications (see table 1) are as follows:

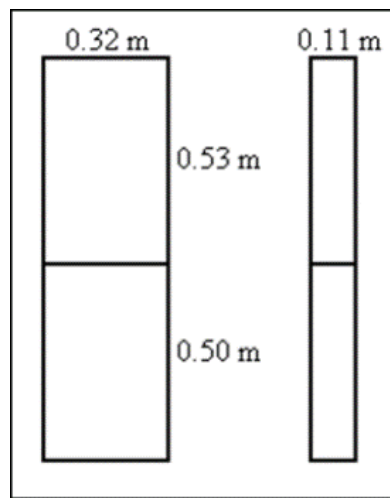
**Table 1** Shake Table Specifications

Dimensions (H x L x W)	61 cm x 46 cm x 13 cm
Total mass	27.2 kg
Payload area (L x W)	46 cm x 46 cm
Maximum payload at 2.5 g	7.5 kg
Maximum travel	± 7.6 cm
Operational bandwidth	10 Hz
Maximum velocity	66.5 cm/s
Maximum acceleration	2.5 g
Lead screw pitch	1.27 cm/rev
Servomotor power	400 W
Amplifier maximum continuous current	12.5 A
Motor maximum torque	7.82 N.m
Lead screw encoder resolution	8192 counts/rev
Effective stage position resolution	1.55 μ m/count
Accelerometer range	± 49 m/s <sup>2</sup>
Accelerometer sensitivity	1.0 g/V

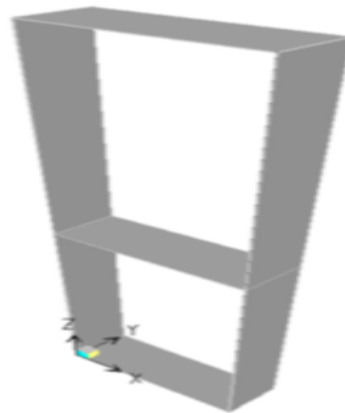


**Figure 2a, b** Illustration of model steel structure and shake table

Model steel structure is 1.03m height. Thickness of elements is 0.001588 m. The structure dimensions are shown in Fig. 3.

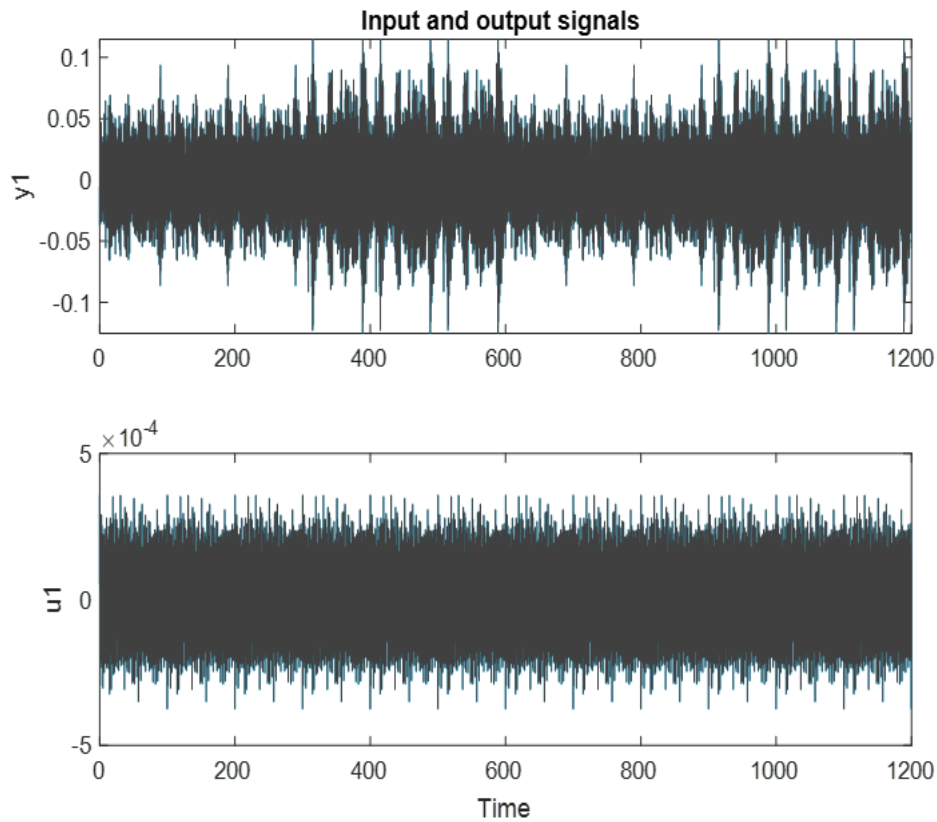


**Figure 3** Illustration of the model steel structure dimensions

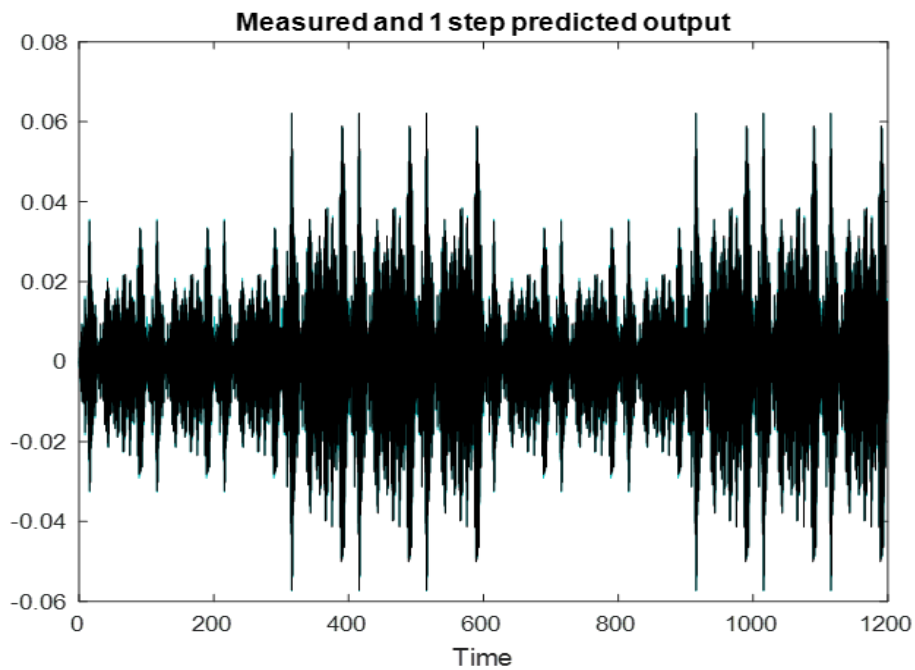


**Figure 4** Finite element model of model steel structure

After analyzing the data in MATLAB using N4SID the following results are summaries in figures 5 - 10.

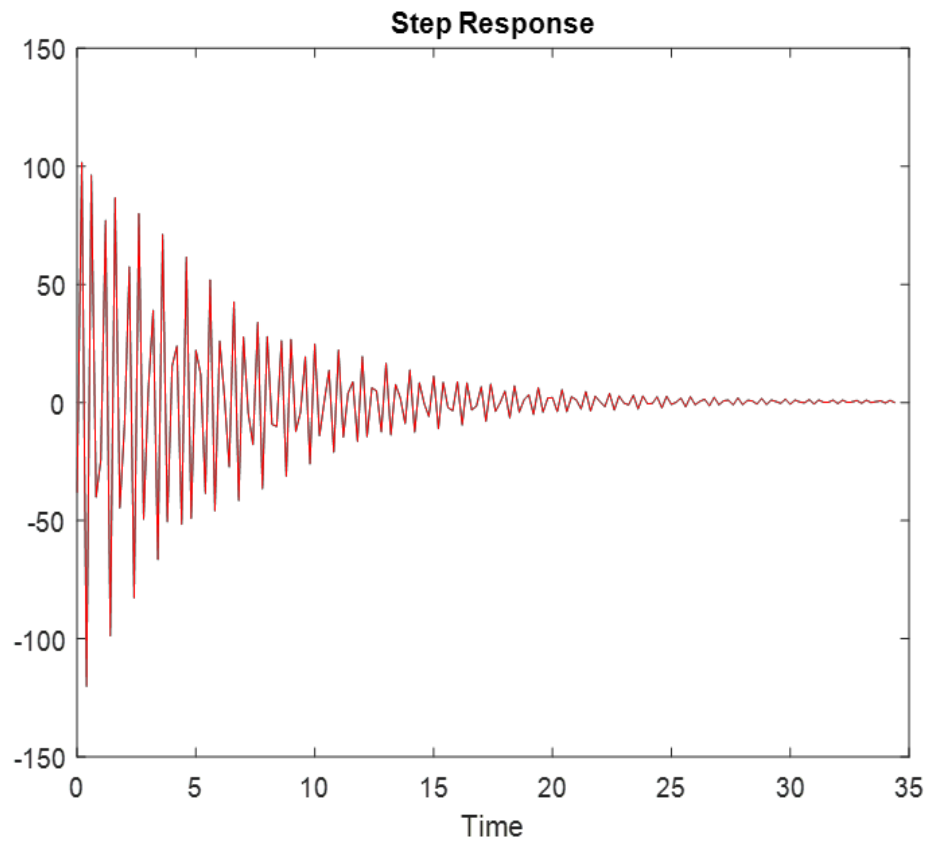


**Figure 5** Input and output signals

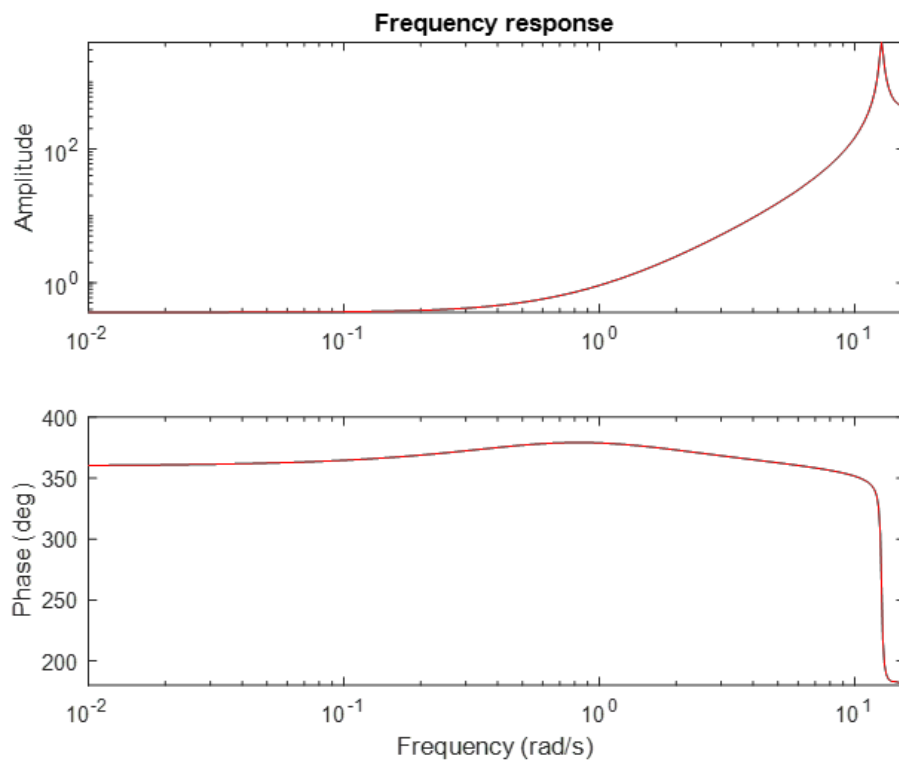


**Figure 6** Model output

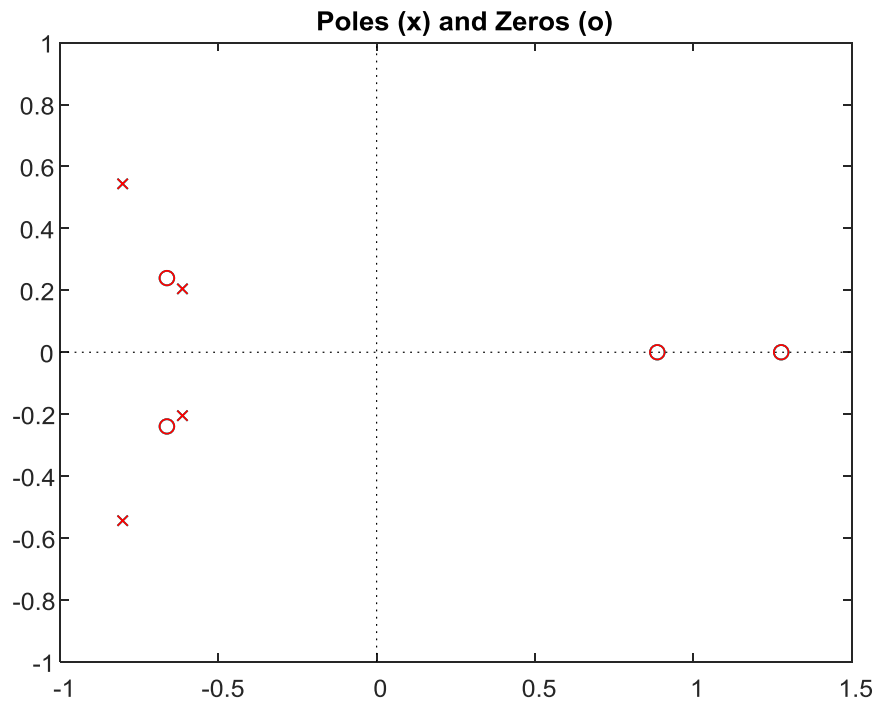
Fit to estimation data 96.44%



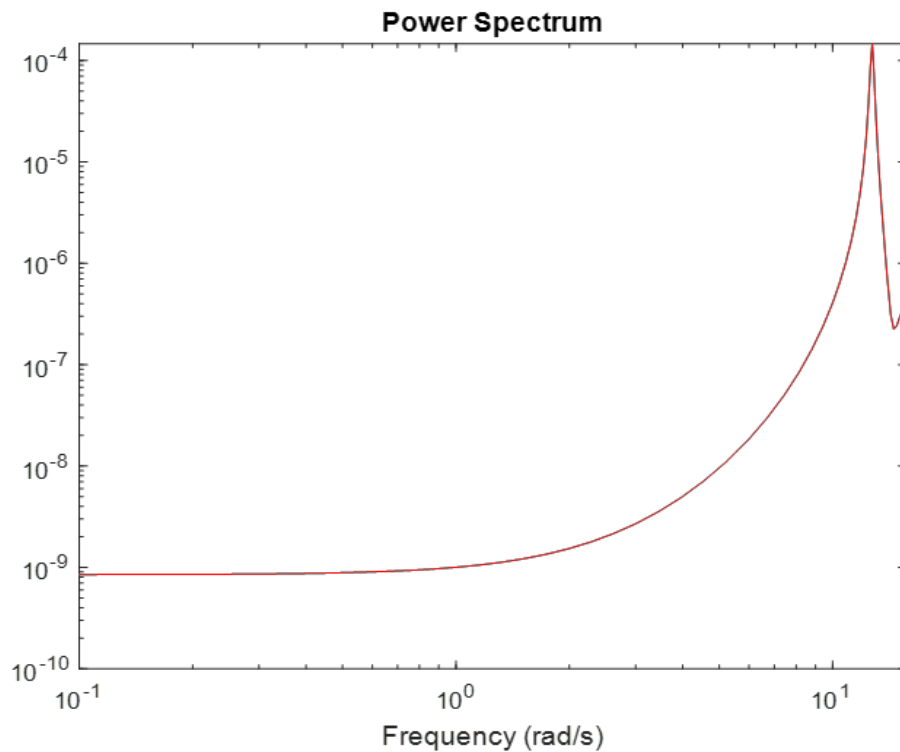
**Figure 7** Transient Response



**Figure 8** Frequency function



**Figure 9** Poles and Zeros



**Figure 10** Noise spectrum

**CONCLUSIONS**

In this paper, a new structural identification tool is proposed to identify the modal properties of structures. Results demonstrated that fit to estimation data was 96.44% and it can be concluded that N4SID system identification method is efficient and accurate in identifying modal data of the structures.



## REFERENCES

1. **G.F. Sirca Jr., H. Adeli**, System identification in structural engineering, *Scientia Iranica A* (2012) 19 (6), 1355–1364.
2. **J. Kim**, System Identification of Civil Engineering Structures through Wireless Structural Monitoring and Subspace System Identification Methods, PhD thesis, University of Michigan, 2011.
3. **A. A. Kasimzade and S. Tuhta**, Stochastic parametric system identification approach for validation of finite element models: industrial Applications, *TWMS Journal of Pure and Applied Mathematics.*, V.3, N.1, 2012, pp.41-61.
4. **P.V. Overschee, B De Moor** , Numerical algorithms for state space subspace system identification, 1993
5. **P.V. Overschee, B De Moor** , Subspace Algorithms for the Identification of Combined Deterministic Stochastic System, *Automatica*, 1994
6. **P.V. Overschee and B. D.Moor**, Subspace identification for linear systems theory-implementation-applications, kluwer academic publishers Boston, London, Dordrecht, 1996.
7. **P Van Overschee, B De Moor, W Favoreel**, Numerical algorithms for subspace state space system identification (N4SID), *ASME Design Engineering*, 1997
8. **TW Flint, RJ Vaccaro**, Performance analysis of N4SID state-space system identification, American Control Conference, 1998
9. MATLAB and Statistics Toolbox Release 2018a, the MathWorks, Inc., Natick, Massachusetts, United States.